

# Quark exchange and quark dynamics in diffractive electroproduction

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A Dyson-Schwinger-based model of pomeron exchange is employed to calculate diffractive  $\rho$ -,  $\phi$ - and  $J/\psi$ -meson electroproduction cross sections. It is shown that the magnitude of the current-quark mass  $m_f$  of the quark and antiquark inside the produced vector meson determines the onset of the asymptotic- $q^2$  power-law behavior of the cross section, and how correlated quark-exchanges are included to provide a complete picture of the diffractive electroproduction of light vector mesons applicable over *all* energies and photon momenta  $q^2$ .

## I. INTRODUCTION

Many aspects of diffractive processes are well described within Regge theory in terms of “Pomeron exchange”. However, since the advent of QCD, the underlying mechanism responsible for pomeron exchange is typically thought to be multiple-gluon exchange. This suggests that experimental investigations of diffractive processes provide a means to study gluon correlations in the non-perturbative, small-momentum-transfer regime of QCD. The additional freedom to vary the momentum transfer  $q^2$  between electron beam and target provides electron beam facilities with more leverage to study pomeron exchange than facilities employing hadron beams. TJ-NAF provides a crucial tool with which to probe the nonperturbative-gluon dynamics underlying pomeron exchange.

In the following, I briefly discuss the elements of a covariant, quantum field theoretic quark-nucleon pomeron-exchange model that was developed in Ref. [1] and describe two applications of this model to diffractive electroproduction. The model employs elements from studies of the Dyson-Schwinger equations of QCD, such as dressed-quark propagators which incorporate confinement, dynamical chiral symmetry breaking, and have the correct asymptotic behavior required by perturbative QCD [2]. The model provides an excellent description of diffractive processes and reveals some aspects of the interplay between perturbative and nonperturbative QCD in these processes. After giving a brief account of the role of the current-quark mass in determining the onset of the asymptotic- $q^2$  behavior of diffractive electroproduction cross sections, I describe how quark exchanges (important at low energies) are included into the model, thereby providing a complete picture of the diffractive electroproduction of light-quark vector mesons over *all* energies and photon-momenta  $q^2$ .

## II. VECTOR MESON ELECTROPRODUCTION

One can study the role of quark dynamics in diffractive processes using a model in which the high-energy interaction between a confined-quark and an on-shell nucleon is given in terms of a pomeron-exchange interaction. In principle, this might be calculated in terms of a multiple-gluon exchange like that shown for  $\rho$ -meson electroproduction in Fig. 1. However, this is unnecessary for the purposes of the present discussion.

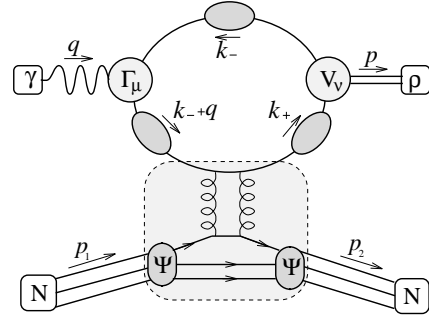


FIG. 1. The shaded box represents the quark-nucleon pomeron-exchange interaction  $\mathcal{G}_\alpha(W^2, t)$  for  $\rho$ -meson electroproduction of Ref. [1]. Within this box is shown one of many possible multiple-gluon exchange diagrams that contributes to the pomeron-exchange interaction  $\mathcal{G}_\alpha(W^2, t)$ .

The  $\rho$ -meson electroproduction current matrix element obtained from the pomeron-exchange model of Ref. [1] is\*

$$\langle p\lambda_\rho; p_2 m' | J_\mu(q) | p_1 m \rangle = 2 t_{\mu\alpha\nu}(q, p) \varepsilon_\nu(p; \lambda_\rho) \bar{u}_{m'}(p_2) \mathcal{G}_\alpha(W^2, t) u_m(p_1), \quad (2.1)$$

where  $u_m(p_1)$  and  $\bar{u}_{m'}(p_2)$  are, respectively, the spinors for the incoming and outgoing nucleon,  $\varepsilon_\nu(p; \lambda_\rho)$  is the polarization vector of the  $\rho$  meson,  $\mathcal{G}_\alpha(W^2, t)$  (represented by a shaded box in Fig. 1) is the model quark-nucleon pomeron-exchange interaction,  $t_{\mu\alpha\nu}(q, p)$  is the photon- $\rho$ -meson transition amplitude (represented by the quark loop in Fig. 1). The quark-nucleon interaction  $\mathcal{G}_\alpha(s, t)$  is given in terms of four parameters: the slope

\* The Euclidean metric  $\delta_{\mu\nu} = \text{diag}(1, 1, 1, 1)$  is employed with *spacelike* momentum  $q_\mu$  satisfying  $q^2 = q_\mu q_\mu > 0$ .

and intercept of the pomeron-exchange Regge trajectory and two coupling constants  $\beta_u$  and  $\beta_s$  for  $u$ - (or  $d$ -) and  $s$ -quarks. Their values are determined in an application to  $\pi N$  and  $KN$  elastic scattering [1].

The photon- $\rho$ -meson transition amplitude is given by

$$t_{\mu\alpha\nu}(q, p) = \beta_u N_c e_0 \text{tr} \int \frac{d^4 k}{(2\pi)^4} S_u(k_{--}) \times \Gamma_\mu(k_{--}, k_{+-}) S_u(k_{+-}) \gamma_\alpha S_u(k_{-+}) V_\nu(k - \frac{1}{2}q; p), \quad (2.2)$$

where  $N_c = 3$ ,  $e_0 = \sqrt{4\pi\alpha_{em}}$  is the elementary charge,  $S_f(k)$  is the quark propagator of flavor  $f$ ,  $\Gamma_\mu(k, k')$  is the quark-photon vertex,  $V_\nu(k; p)$  is the  $\rho$ -meson Bethe-Salpeter amplitude,  $\gamma_\alpha$  is a Dirac matrix, and the trace is over Dirac indices.

Evaluation of  $t_{\mu\alpha\nu}(q, p)$  requires explicit forms for the Schwinger functions  $S_f(k)$ ,  $\Gamma_\mu(k, k')$ , and  $V_\nu(k, p)$  in Eq. (2.2). These are taken from phenomenological studies of hadron observables based on the Dyson-Schwinger equations of QCD.

The most general quark-photon vertex  $\Gamma_\mu(k, k') = \Gamma_\mu^T(k, k') + \Gamma_\mu^{BC}(k, k')$ , where  $\Gamma_\mu^T(k, k')$  is transverse to the photon momentum  $q_\mu = (k - k')_\mu$  and  $\Gamma_\mu^{BC}(k, k')$  is *completely determined* from the Ward-Takahashi identity and the dressed quark propagator  $S_f(k)$ . Phenomenological studies find that the transverse contributions to the vertex in  $\Gamma_\mu^T(k, k')$  are unimportant for the calculation of observables for spacelike photon momenta. Hence, it is reasonable in the present study to neglect this term and take the dressed quark-photon vertex as  $\Gamma_\mu^{BC}(k, k')$ . This entails a parameter-free nonperturbatively dressed quark-photon vertex that is consistent with the dressed-quark propagator  $S_f(k)$ .

In a general covariant gauge, the dressed-quark propagator is written as  $S_f(k) = -i\gamma \cdot k \sigma_V^f(k^2) + \sigma_S^f(k^2)$ . Numerical studies [3] of the Dyson-Schwinger equations find that the essential features of the dressed-quark propagator are well represented by a simple parametrization:

$$\begin{aligned} \bar{\sigma}_S^f(x) &= (b_0^f + b_2^f \mathcal{F}[\epsilon x]) \mathcal{F}[b_1^f x] \mathcal{F}[b_3^f x] \\ &\quad + 2\bar{m}_f \mathcal{F}[2(x + \bar{m}_f^2)] + C_{m_f} e^{-2x}, \\ \bar{\sigma}_V^f(x) &= \frac{2(x + \bar{m}_f^2) - 1 + e^{-2(x + \bar{m}_f^2)}}{2(x + \bar{m}_f^2)^2}, \end{aligned} \quad (2.3)$$

where  $\mathcal{F}[x] = (1 - e^{-x})/x$ ,  $x = k^2/\lambda^2$ ,  $\bar{\sigma}_S^f = \lambda \sigma_S^f$ ,  $\bar{\sigma}_V^f = \lambda^2 \sigma_V^f$ ,  $\bar{m}_f = m_f/\lambda$ ,  $\lambda = 0.566$  GeV, and  $\epsilon = 10^{-4}$ . This dressed-quark propagator has *no* Lehmann representation and hence describes the propagation of a confined quark. Furthermore, it reduces to a bare-fermion propagator with current-quark mass  $m_f$  for large, space-like momenta, in accordance with perturbative QCD. The parameters for the  $u$ -,  $d$  and  $s$ -quark propagators were determined in Ref. [2] by performing a  $\chi^2$  fit to a range of  $\pi$ - and  $K$ -meson observables.

The final element is the  $\rho$ -meson BS amplitude  $V_\nu(k; p)$  which was modeled in Ref. [1] as the sum of exponential

and monopole functions in the relative quark-antiquark momentum  $k^2$ . The parameters were determined by requiring the BS amplitude leads to the experimental values for the  $\rho \rightarrow \pi\pi$  and  $\rho \rightarrow e^+e^-$  decay widths.

Having determined the Schwinger functions in Eq. (2.2) from Dyson-Schwinger studies of low-energy hadron observables, and the quark-nucleon pomeron-exchange interaction  $\mathcal{G}_\alpha(W^2, t)$  from meson-nucleon elastic scattering, one can use Eqs. (2.1) and (2.2) to calculate the  $\rho$ -meson electroproduction cross section. The result (solid curve in Fig. 2) is in excellent agreement with the data for all  $q^2$ . One important feature of the quark-nucleon interaction  $\mathcal{G}_\alpha(W^2, t)$  in Eq. (2.1) is that it is independent of  $q^2$ . Hence, all of the  $q^2$  dependence in diffractive electroproduction cross sections arises from the Schwinger functions in the quark-loop integration of Eq. (2.2). The  $q^2$  dependence of the electroproduction cross section, shown in Fig. 1, results from having employed elementary Schwinger functions that describe low-energy meson observables such as the  $\pi$ - and  $K$ -meson electromagnetic form factors.

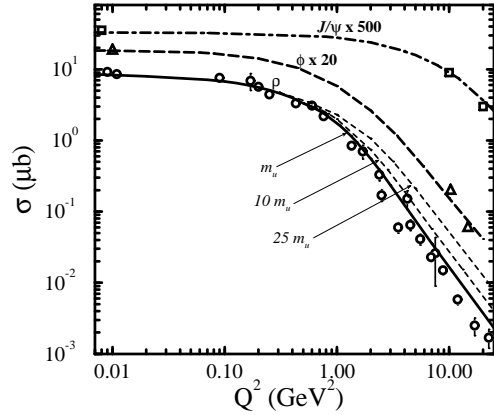


FIG. 2. The  $\rho^0$ -,  $\phi$ -, and  $J/\psi$ -meson electroproduction cross sections at  $W = 15$ , 100 and 100 GeV, respectively. (The latter two results and corresponding data are rescaled by amounts indicated.) The narrow dashed curves show  $\rho$ -meson electroproduction results obtained using fictitious current-quark masses of 10 and 25 times larger than  $m_u = 5.5$  MeV.

In the study of Ref. [1], the role of the current-quark mass  $m_u$  in determining the  $q^2$  dependence of electroproduction was explored using fictitious current quark masses  $m_f$  that are 10 and 25 times greater than  $m_u = 5.5$  MeV to recalculate the  $\rho$ -meson electroproduction cross section. The results are shown in Fig. 2. A comparison of these two curves to the result obtained using the correct value of  $m_u = 5.5$  MeV, suggests two important features of diffractive electroproduction. First, the magnitude of the photoproduction cross section ( $q^2 = 0$ ) is unaffected by changes to  $m_u$ . This is because the quark-photon vertex  $\Gamma_\mu^{BC}(k, k')$  satisfies the Ward Identity which, when coupled with the normalized  $\rho$ -meson

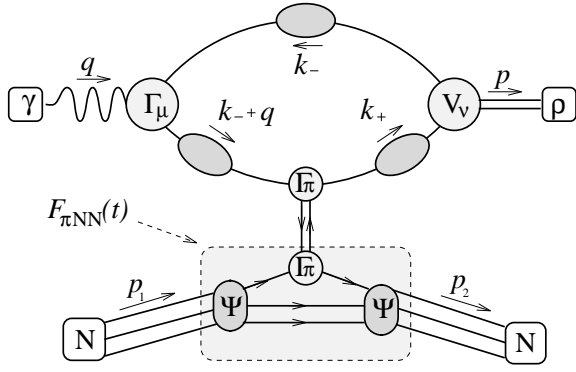


FIG. 3. The  $\rho$ -meson electroproduction current due to  $t$ -channel effective- $\pi$  exchange. The shaded box represents the effective- $\pi NN$  form factor,  $F_{\pi NN}(t)$ .

BS amplitude, tightly constrains the magnitude of the cross section at  $q^2 = 0$ . Second, although all three curves converge in the photoproduction limit of  $q^2 \rightarrow 0$ , they diverge significantly for larger values of  $q^2$ . Ultimately, these curves exhibit the same  $q^{-4}$  behavior, but the onset of this asymptotic behavior is *determined* by the magnitude of  $m_u$ ; that is, a larger current-quark mass for the quark and antiquark inside the produced vector meson postpones the onset of the asymptotic power-law behavior until a larger value of  $q^2$ . The results for  $\phi$  and  $J/\psi$  electroproduction obtained from the model of Ref. [1] are shown in Fig. 2. Both the model calculation and experimental data exhibit the anticipated behavior.

The dependence of diffractive electroproduction on the current-quark mass  $m_f$  provides an explanation of the dramatic result that although the  $\rho$ -meson electroproduction cross section is *two orders of magnitude* larger than that of the  $J/\psi$  meson near  $q^2 = 0$ , they are nearly equal for larger  $q^2$ . This behavior is a result of having used the dressed-quark propagator  $S_f(k)$  from Dyson-Schwinger studies which evolve dynamically with the quark momentum  $k^2$ . For example, the behavior of the  $u$ -quark propagator  $S_u(k)$  at small momentum  $k^2$  is characterized by constituent-quark-like mass scales ( $\approx 330$  MeV) while at larger momentum, it is characterized by the current-quark mass  $m_u = 5.5$  MeV. The dynamical evolution of mass scales in the dressed-quark propagator  $S_f(k)$  is essential for the description of electroproduction at all  $q^2$ .

### III. PHOTO-MESON TRANSITION AMPLITUDES

At lower energies, both pomeron and correlated-quark exchanges are important. The energy at which pomeron exchange and effective-meson exchanges each contribute about a half of the photoproduction cross section is  $W \approx 6$  GeV for  $\rho$  mesons and  $W \approx 3$  GeV for  $\phi$  mesons. (See Fig. 5.) These are incorporated into the pomeron-exchange model by a reorganization of the nonlocal inter-

actions between quarks into sums of exchanges of effective fields with mesonic quantum numbers, referred to as effective-meson fields. An account of how this procedure is carried out in theory and practice is given in Ref. [4].

As an example, consider the contribution to  $\rho$ -meson electroproduction due to the  $t$ -channel exchange of an effective “ $\pi$  meson”. Applying the techniques described in Ref. [4], one obtains the electroproduction current element:

$$\langle p\lambda_\rho; p_2 m'_s | J_\mu(q) | p_1 m_s \rangle = \Lambda_{\mu\nu}(q, p) \varepsilon_\nu(p; \lambda_\rho) \times \frac{1}{m_\pi^2 - t} \bar{u}(p_2, m'_s) \gamma_5 F_{\pi NN}(t) u(p_1, m_s), \quad (3.1)$$

where  $\Lambda_{\mu\nu}(q, p)$  is the photon- $\rho$ -meson transition amplitude due to the exchange of an effective  $\pi$  meson (denoted in Fig. 3 by a quark loop) and  $F_{\pi NN}(t) = (1 - t/\Lambda^2)^{-1}$  with  $\Lambda \approx 1$  GeV is the effective- $\pi$ - $NN$  form factor (denoted by a shaded box in Fig. 3).

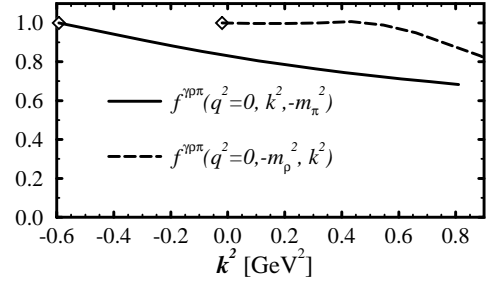


FIG. 4. The off-shell  $\gamma\rho\pi$  transition form factor is shown for the cases when either the  $\rho$  or  $\pi$  meson is off shell. Diamonds indicate the point at which all three particles are on shell. In a calculation of  $\rho$ -meson photoproduction the  $\rho$ -meson is on-shell so that only the latter form factor (dashed curve) is needed.

The photo- $\rho$ -meson transition amplitude  $\Lambda_{\mu\nu}(q, p)$  is given by

$$\Lambda_{\mu\nu}(q, p) = \frac{e_0}{\sqrt{Z_\pi(t)}} \text{tr} \int \frac{d^4 k}{(2\pi)^2} S_u(k_-) \Gamma_\mu(k_-, k_- + q) \times S_u(k_- + q) V_\nu(k; p) S_u(k_+) \Gamma_\pi(k + \frac{1}{2}q; -t), \quad (3.2)$$

where  $\Gamma_\pi(k; p)$  is the on-shell pion BS amplitude, and  $Z_\pi(t)$  is the factor appearing in the effective-meson “propagator”  $\Delta_{\text{eff}}(t) = Z_\pi^{-1}(t) (m_\pi^2 - t)^{-1}$  which arises from the nonlocal nature of the effective pion. The factor  $Z_\pi(t)$  is calculated in a straight-forward manner from the  $\pi$ -meson BS amplitude  $\Gamma_\pi(k; p)$  and the dressed-quark propagator  $S_f(k)$ , as described in Ref. [4]. Considerations of parity and Lorentz covariance allow one to rewrite the amplitude as

$$\Lambda_{\mu\nu}(q, p) = \frac{e_0}{m_\rho} g_{\gamma\rho\pi} \epsilon_{\mu\nu\alpha\beta} q_\alpha p_\beta f^{\gamma\rho\pi}(q^2, p^2, -t), \quad (3.3)$$

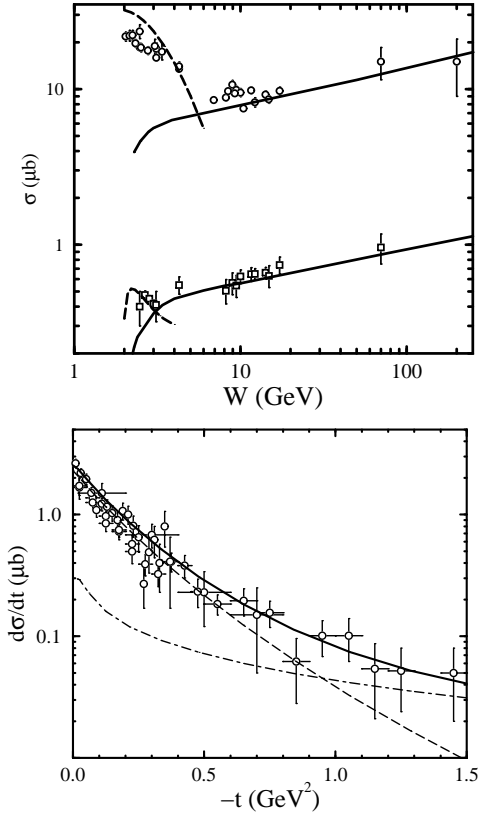


FIG. 5. Top: Contributions to  $\rho$ - and  $\phi$ -meson photoproduction from pomeron exchange (solid) and meson exchanges (dashed). Bottom: The differential cross section for  $\phi$ -meson photoproduction ( $q^2 = 0$ ) for  $3.0 \leq W \leq 3.5$  GeV. The contributions due to the pomeron exchange (dashed), and  $\pi$  and  $\eta$  exchanges (dot-dashed) and their sum (solid) are shown.

where  $g_{\gamma\rho\pi}$  is the  $\rho \rightarrow \gamma\pi$  decay constant,  $\epsilon_{\mu\nu\alpha\beta}$  is the usual Levi-Cevita tensor, and  $f^{\gamma\rho\pi}(q^2, p^2, -t)$  is the  $\gamma\rho\pi$ -transition form factor, defined so that when all three particles are on their mass shell,  $f^{\gamma\rho\pi}(q^2 = 0, p^2 = -m_\rho^2, -t = -m_\pi^2) = 1$ . All of the elements required to calculate Eq. (3.1) are known. The resulting photo- $\rho$ -meson transition form factor is shown in Fig. 4 for a range of off-shell  $\pi$ - and  $\rho$ -meson momenta.

The slow fall off of the form factors shown in Fig. 4 is typical of photo-meson transition amplitudes. It is a result of the fact that the factor  $\sqrt{Z(t)}$  decreases with  $-t$ , which tends to weaken and partially counteract the rapid fall off usually observed in spacelike transition form factors when all mesons are on shell. This behavior has also been observed in studies of other photo-meson transition amplitudes, such as  $\gamma\gamma\pi$  and  $\gamma\pi \rightarrow \pi\pi$  [5].

With these meson-transition form factors, one can use the quark-nucleon pomeron-exchange model to calculate  $\rho$ - and  $\phi$ -meson electroproduction at all energies and  $q^2$ . This work is currently in progress. However, shown in the top of Fig. 5 are the cross sections for  $\rho$ - and  $\phi$ -meson production with  $f^{\gamma\rho\pi}(q^2 = 0, p^2 = -m_\rho^2, -t) = 1$ . Hence,

these predictions should overestimate the contributions from meson exchanges. In the bottom plot of Fig. 5, are results for the  $\phi$ -meson differential cross section at  $W = 3$  GeV. At moderate  $t$ , the meson-exchange contributions (from  $\pi$  and  $\eta$  exchange) overwhelm that of pomeron-exchange. At lower energies, meson exchanges become increasingly more important and one must include them (and their respective transition form factors) to obtain good agreement with the data, even at small  $t$ .

#### IV. CONCLUSION

I have given a brief outline of how quark dynamics and correlated-quark exchanges may be explored in diffractive electroproduction using a quantum field theoretic framework based on the Dyson-Schwinger equations of QCD.

In numerical studies of the Dyson-Schwinger equations, both perturbative and nonperturbative aspects of QCD are manifest in the solutions obtained for the elementary Schwinger functions, such as the dressed-quark propagator and meson Bethe-Salpeter amplitudes. In phenomenological applications, such as the one described here, one explores the consequences of employing such dressed Schwinger functions and their effect on experimental observables. In this way, one is able to probe the underlying dynamics of quarks and gluons involved in exclusive processes and further improve our understanding of QCD.

#### ACKNOWLEDGMENTS

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- [1] M.A. Pichowsky and T.-S.H. Lee, Phys. Rev. D **56**, 1644 (1997).
  - [2] C.J. Burden, C.D. Roberts and M.J. Thomson, Phys. Lett. B **371**, 163 (1996).
  - [3] P. Maris and C.D. Roberts, Phys. Rev. C **56**, 3369 (1997); P. Maris and C.D. Roberts, *Pseudovector components of the pion,  $\pi^0 \rightarrow \gamma\gamma$ , and  $F_\pi(q^2)$* , LANL e-print # nucl-th/9804062, (1998).
  - [4] P.C. Tandy, Prog. Part. Nucl. Phys. **39**, 117 (1997).
  - [5] M.R. Frank, K.L. Mitchell, C.D. Roberts and P.C. Tandy, Phys. Lett. B **359**, 17 (1995); R. Alkofer and C.D. Roberts, Phys. Lett. B **369**, 101 (1996).